ON THE INTERRELATION OF TWO BASIC PROBLEMS IN THE THEORY OF FLOW WHICH CONSIDERS RESIDUAL MICROSTRESSES

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Two basic problems in the theory of flow are the determination of the loading path for a given deformation path, and, conversely, the determination of the deformation path for a given loading path.

Below it is shown that in the theory of flow that considers residual microstresses [1] there exists a close mathematical interrelation between these two problems in the case of linear hardening and a linear Bauschinger effect. The possibility of solving one of them (for example, when a definite loading path is specified) implies the possibility of solving the other (for an analogous deformation path).

1. Let a material under uniaxial loading have linear hardening and a linear Bauschinger effect*

$$\begin{aligned} \varepsilon_x^{*p} &= A\,(\sigma_x' - \sigma_T') \qquad (\varepsilon_x^{*p} = 2G\varepsilon_x^{p}, \quad A = \text{const}) \\ S_x &= \alpha\,(\sigma_x' - \sigma_T') \qquad (0 \leqslant \alpha \leqslant 1, \alpha = \text{const}) \end{aligned} \tag{1}$$

Then the equations of the theory [1] may be brought to the following form:

$$d\sigma_{ij}' = d\overset{\circ}{\sigma}{}_{ij}' + \frac{\alpha}{1-\alpha}\overset{\circ}{\sigma}{}_{ij}' \frac{dT^\circ}{T^\circ} \tag{2}$$

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* The notation of [1] is used below.

$$\varepsilon_{ij}^{*p} = \frac{A}{\alpha}\,(\sigma_{ij}^{\circ\prime} - \sigma_{ij}^{\prime})\ (\varepsilon_{ij}^{*p} = 2G\varepsilon_{ij}^{p}) \tag{3}$$

If the loading path is specified, then the tensor $\sigma_{ij}^{0\prime}$ is first determined from the solution of equation (2), after which the tensor ε_{ij}^{p} is determined from formula (3).

Likewise, if the deformation path is specified, then it is sufficient to examine the following group of equations:

$$d\varepsilon_{ij}^{*\prime} = d\sigma_{ij}^{\circ\prime} + \frac{k_0}{1-k_0}\sigma_{ij}^{\circ\prime}\frac{dT^\circ}{T^\circ}, \qquad k_0 = \frac{\alpha + A}{1 + A} \tag{4}$$

$$\sigma_{ij}^{\prime} = \frac{\alpha\varepsilon_{ij}^{*\prime} + A\sigma_{ij}^{\circ\prime}}{\alpha + A}, \qquad \varepsilon_{ij}^{*\prime} = \sigma_{ij}^{\prime} + \varepsilon_{ij}^{*p} \tag{5}$$

The tensor $\sigma_{ij}^{0\prime}$ is determined from equations (4), and the tensor σ_{ij} from equations (5).

The following conclusions follow from a comparison of (2) and (4).

a) The problem of finding the stresses for a specified deformation path (for a certain value α) is equivalent to the problem of finding strains for a specified loading path and a certain reduced value $\alpha = k_0$.

b) If $\alpha = 1$, then $k_0 = 1$, i.e in the case of an ideal Bauschinger effect, there is a complete analogy between the two problems ($\alpha = 1$, $T^\circ = \text{const}$).

c) In the Lening theory of flow, the problem of finding the deformation path is simpler than the problem of finding the loading path, since $k_0 = A/1 + A$ for $\alpha = 0$. As is seen from (2) and (4), in the general case both problems are of the same degree of difficulty. Moreover, for weakly strain-hardening materials one may take $k_0 \sim 1$, since $A_0 \gg 1$. That is, as the first approximation for determining the auxiliary tensor σ_{ij}^{0} for a specified deformation path one may use the theory for an ideal Bauschinger effect. It is interesting that the greatest error in this case is obtained in the Lening flow theory.

We note that in the latter theory the first approximation (for a given deformation path) passes over into the well-known Reuss theory.

2. In [1], the strain-hardening coefficient is determined from the condition that

$$\int \sigma_{ij}^{\circ\prime}\, d\varepsilon_{ij}^{p}$$

is a function that depends only on $\sigma_{ij}^{0\prime}\sigma_{ij}^{0\prime}$. In addition, it is possible by generalizing the results introduced already in [2] to compute the quantity $(\varepsilon_{ij}^{*\prime} - \sigma_{ij}^{0\prime})(\varepsilon_{ij}^{*\prime} - \sigma_{ij}^{0\prime})$, which depends only on $\sigma_{ij}^{0\prime}\sigma_{ij}^{0\prime}$. In this, naturally, the equivalence of the two problems is retained in the case of a linear Bauschinger effect and linear hardening

$$\sigma_{ij}^{\circ\prime} d\sigma_{km}^{\prime} - \sigma_{km}^{\circ\prime} d\sigma_{ij}^{\prime} = \sigma_{ij}^{\circ\prime} d\sigma_{km}^{\circ\prime} - \sigma_{km}^{\circ\prime} d\sigma_{ij}^{\prime} \qquad (6)$$

$$T^{\circ} = T_0 + \frac{1-\alpha}{\alpha} T_s$$

$$T_s^{\,2} = \frac{1}{2} S_{ij} S_{ij} = \frac{1}{2} (\sigma_{ij}^{\prime} - \sigma_{ij}^{\circ\prime})(\sigma_{ij}^{\prime} - \sigma_{ij}^{\circ\prime})$$

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